



Structural Bioinformatics

GENOME 541

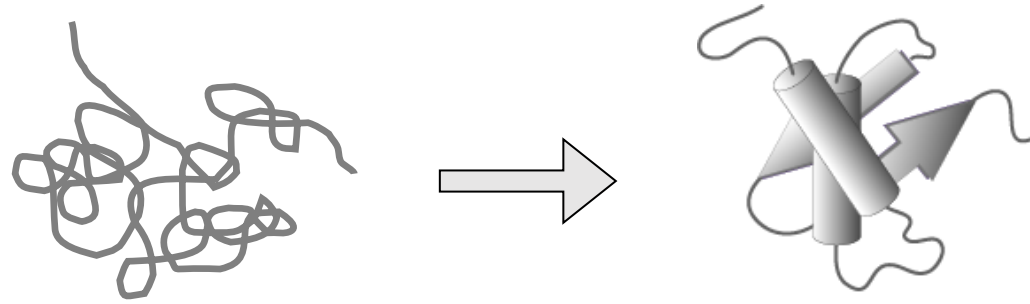
Spring 2023

Lecture 2: Biomolecular

Energy Functions

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Protein Folding

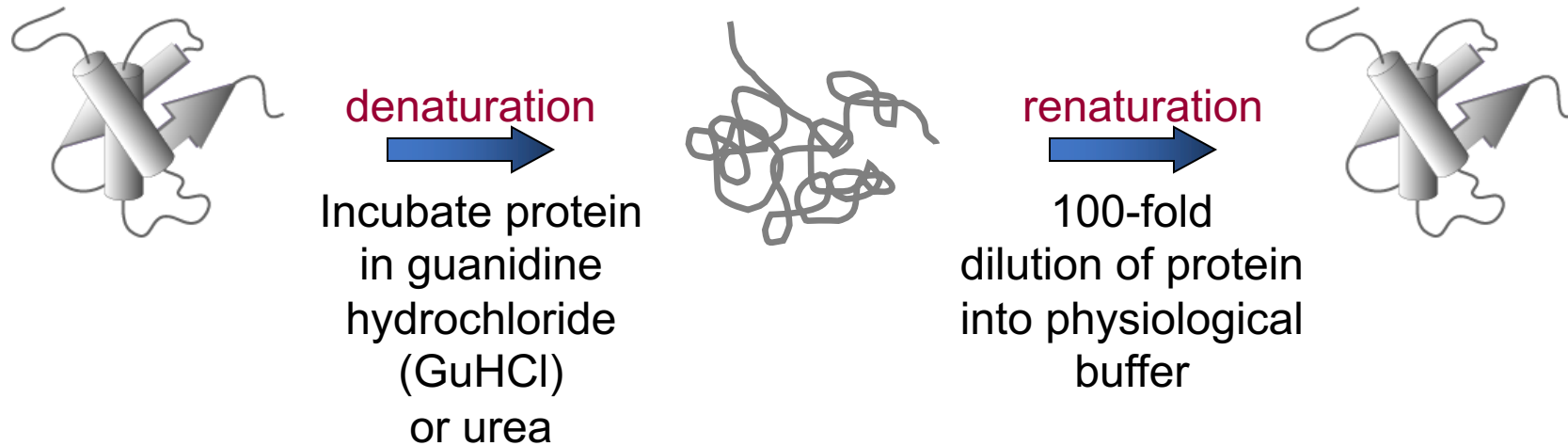


The process by which a protein goes from being an unfolded polymer with no activity to a uniquely structured and active protein.

Why do we care about protein folding?

- Understanding how protein's folds informs us of *sequence to structure* mapping
- Protein misfolding has been implicated in many human diseases (e.g. Alzheimer's, Parkinson's)

Protein folding *in vitro* is often reversible



- **the amino acid sequence of a polypeptide is sufficient to specify its three-dimensional conformation**
- protein folding is a spontaneous process that does not require the assistance of extraneous factors

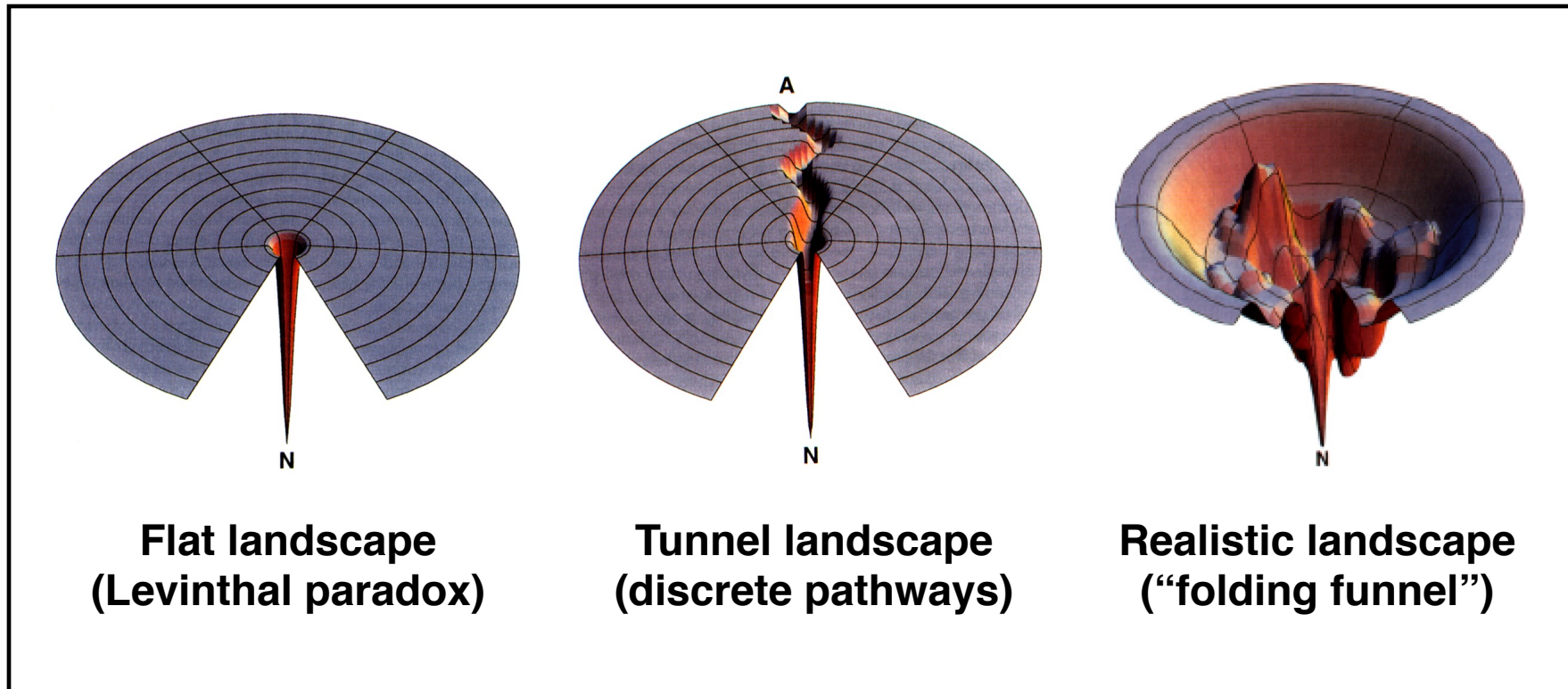
Anfinsen, CB (1973) Principles that govern the folding of protein chains. *Science* **181**, 223-230.

How Do Proteins Fold?

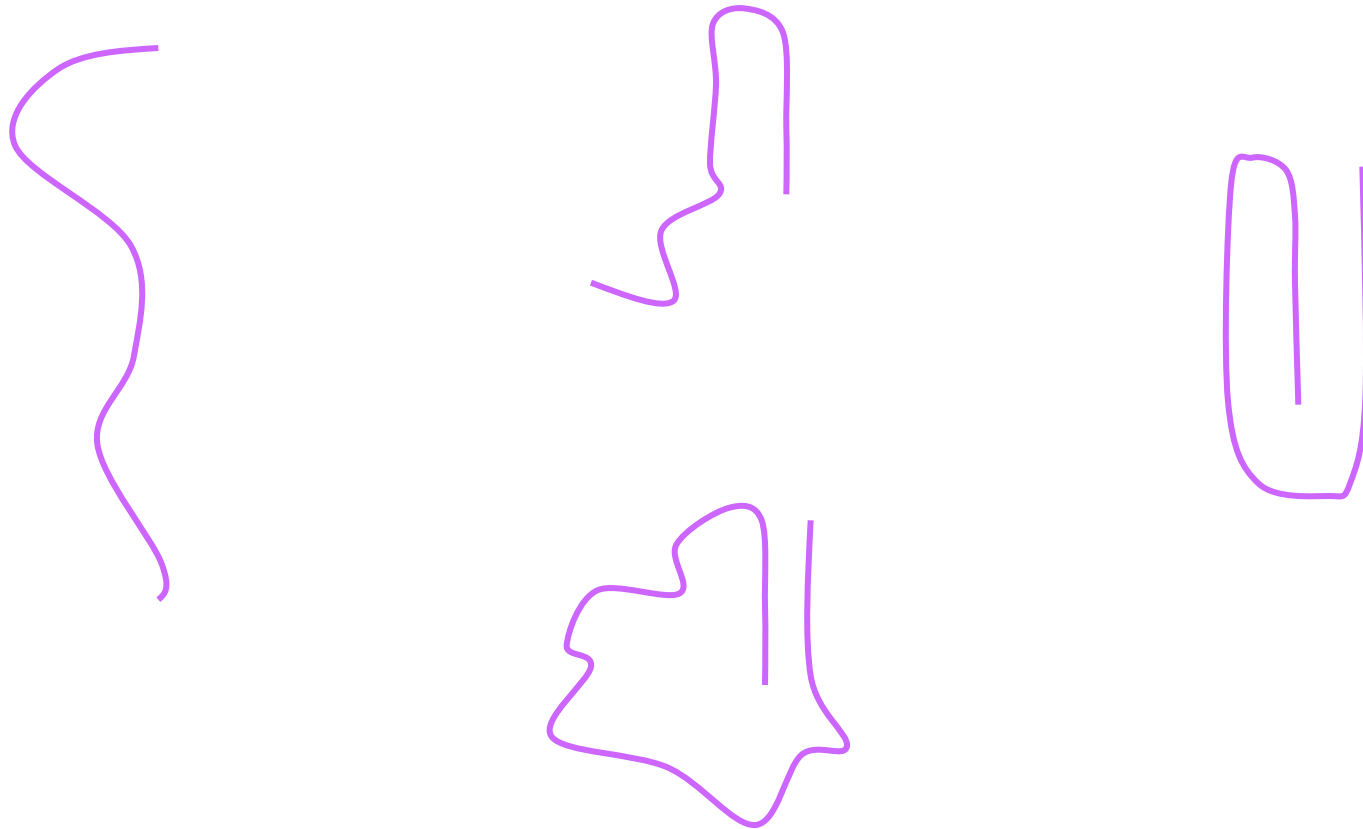
- Cyrus Levinthal tried to estimate how long it would take a protein to do a random search of conformational space for the native fold.
- Imagine a 100-residue protein with three possible conformations per residue. Thus, the number of possible folds = $3^{100} = 5 \times 10^{47}$.
- Let us assume that protein can explore new conformations at the same rate that bonds can reorient (10^{13} structures/second).
- Thus, the time to explore all of conformational space = $5 \times 10^{47} / 10^{13} = 5 \times 10^{34}$ seconds = 1.6×10^{27} years \gg age of universe
- This is known as the **Levinthal paradox**.

How do proteins fold?

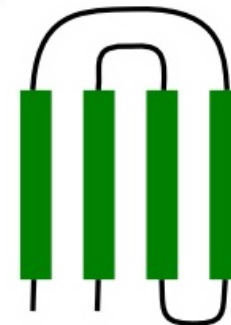
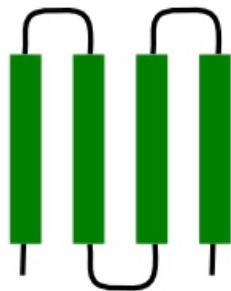
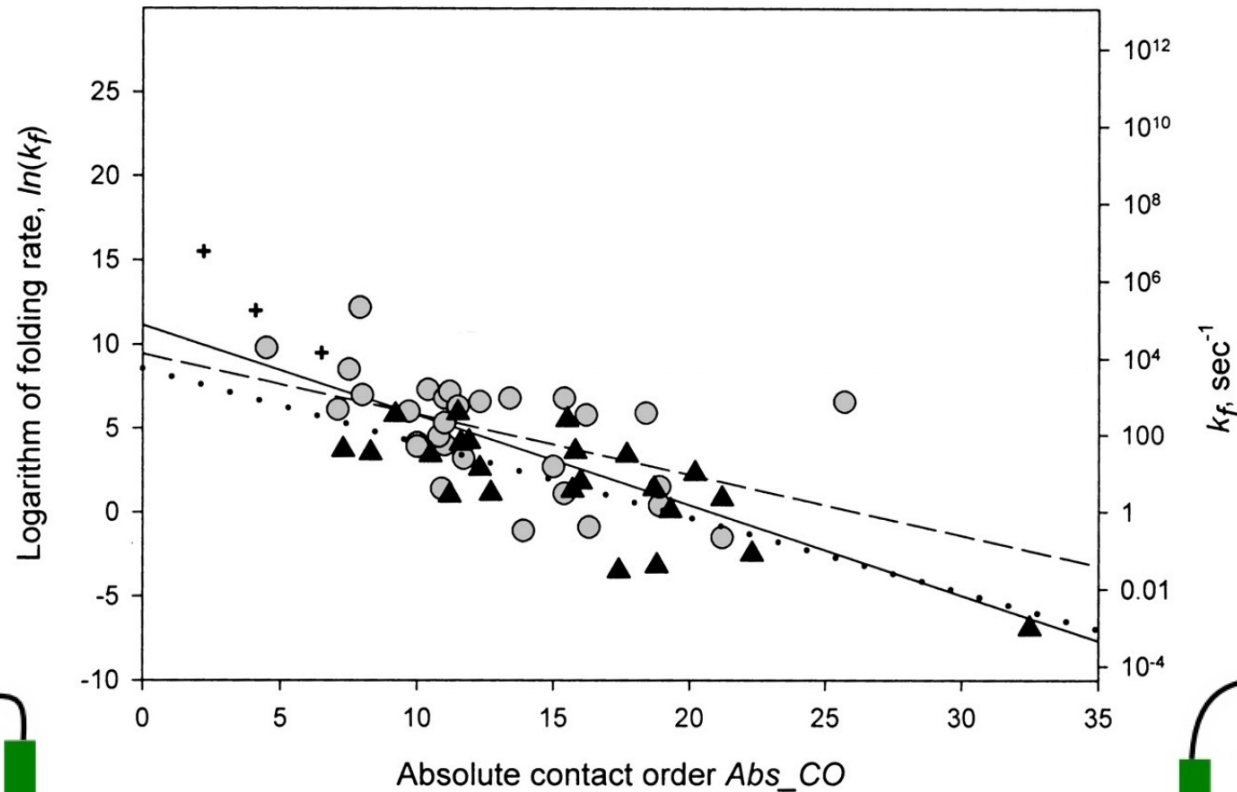
Do proteins fold by a very discrete pathway?



Do certain portions of a protein fold first?



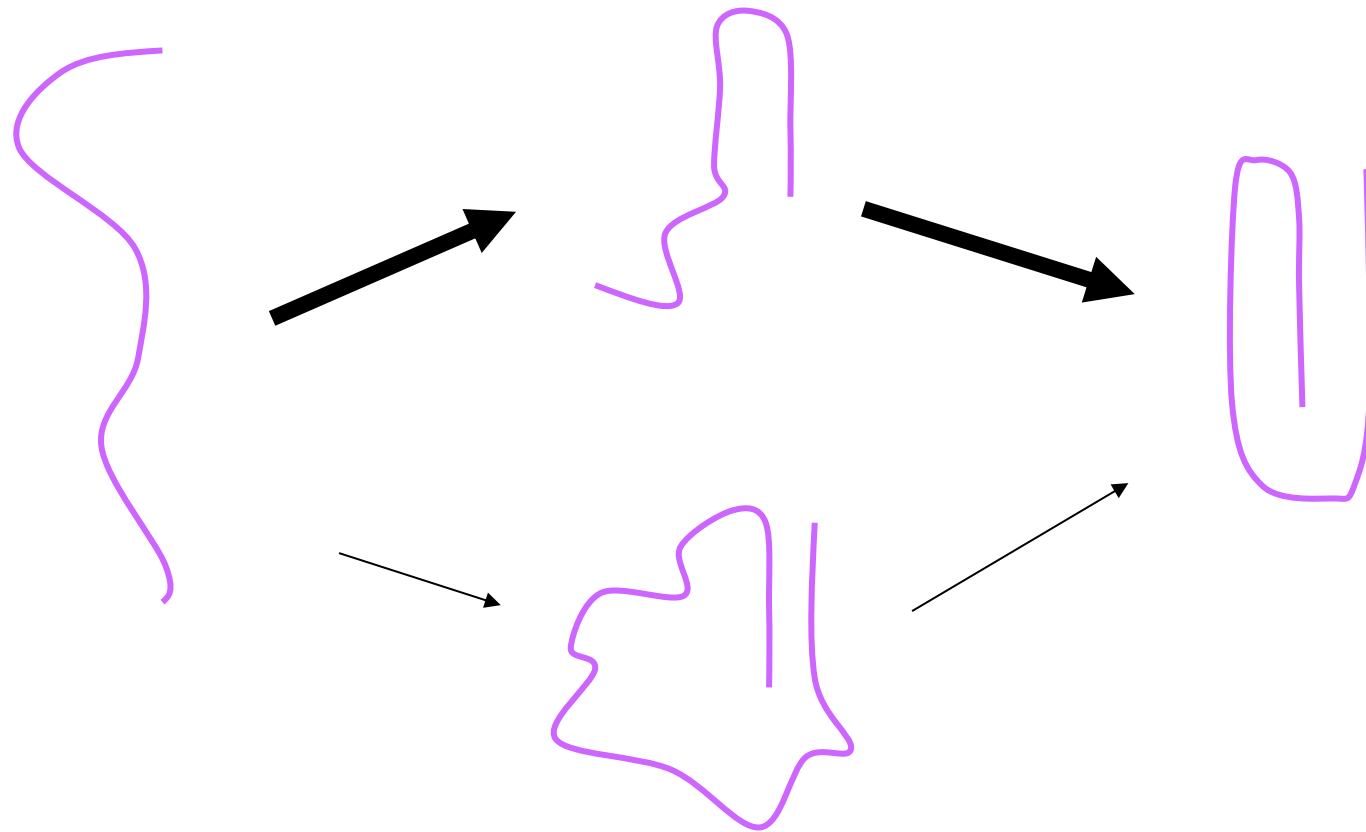
Protein folding rates correspond with contact order



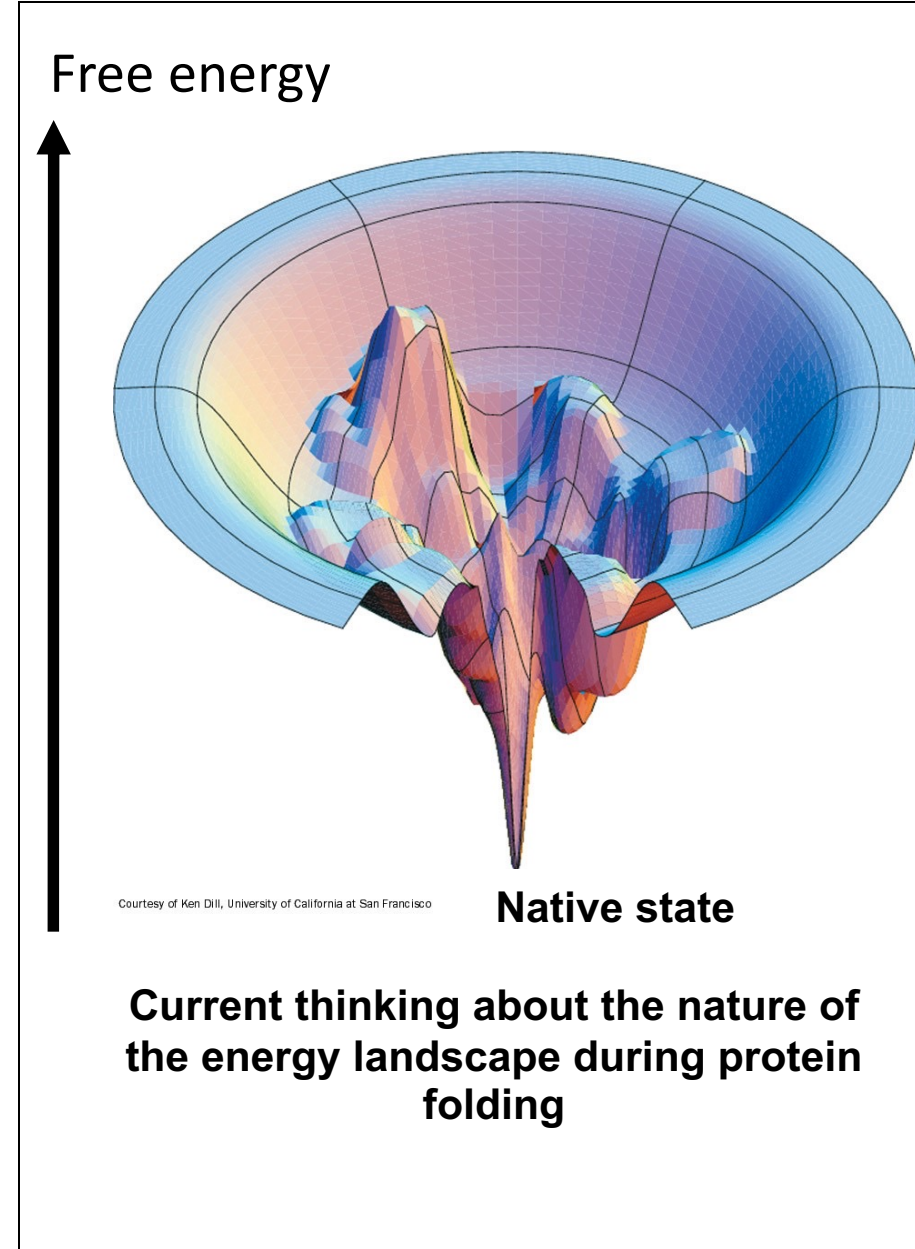
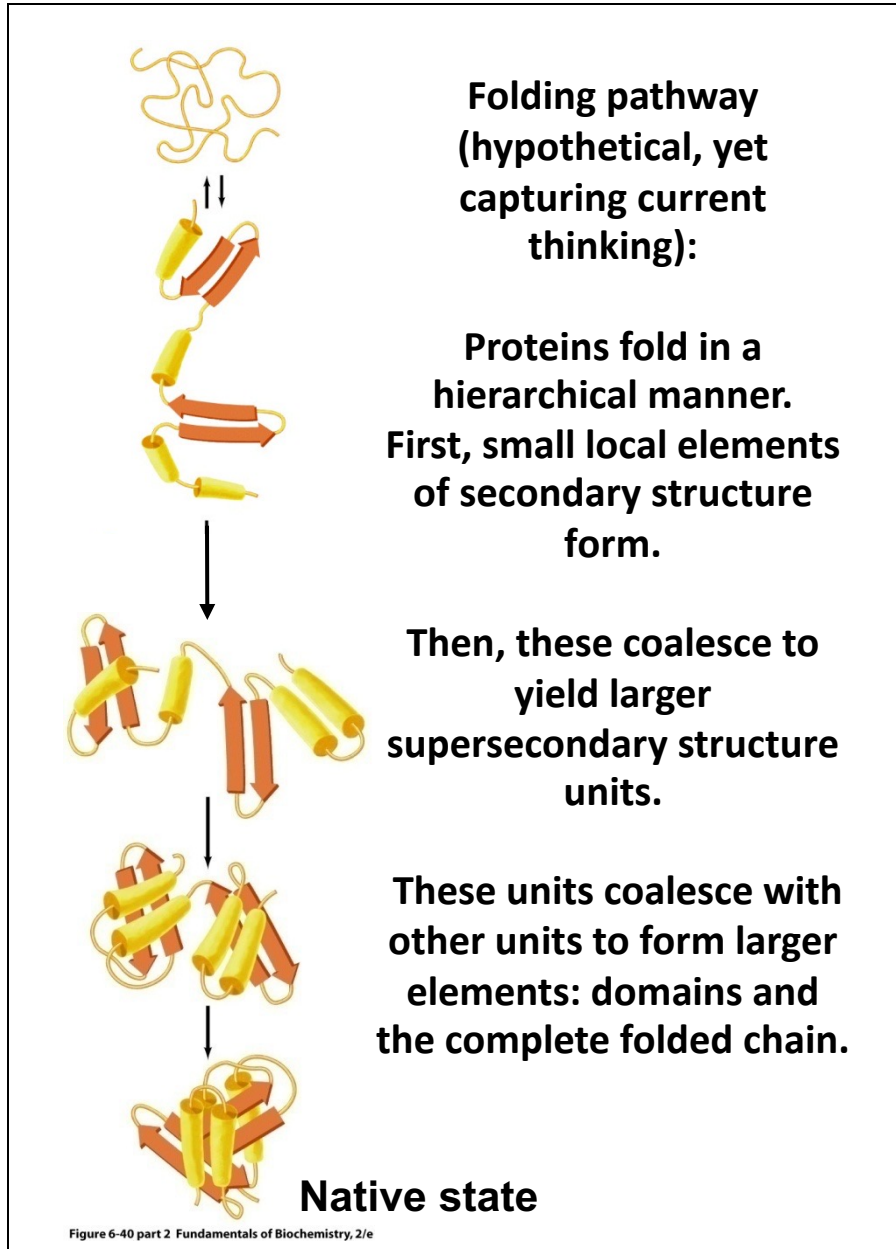
$$CO = \frac{1}{L \cdot N} \sum^N \Delta S_{i,j}$$

Do certain portions of a protein fold first?

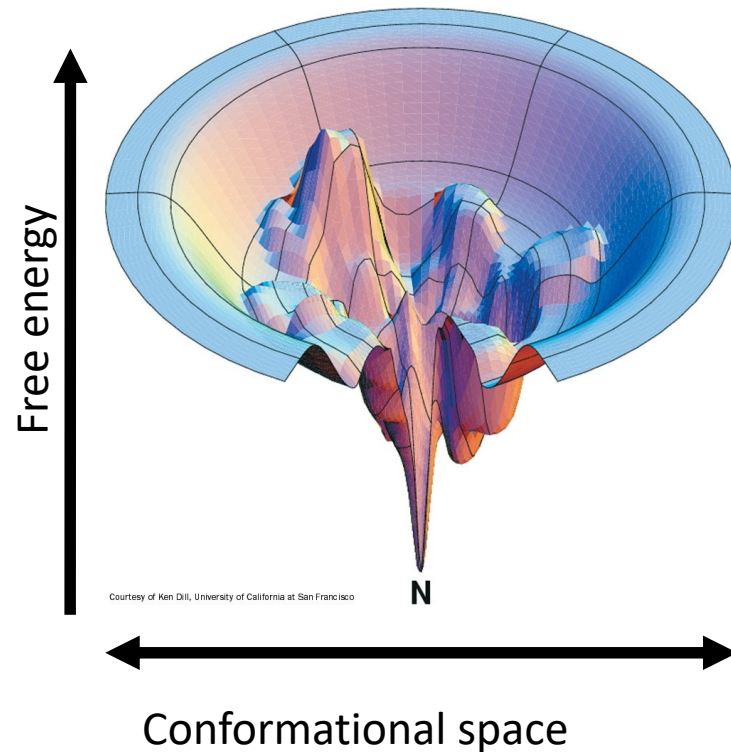
Interactions between residues *close to each other along the polypeptide chain* are more likely to form early in folding.



Folding pathways and energy landscapes in protein folding

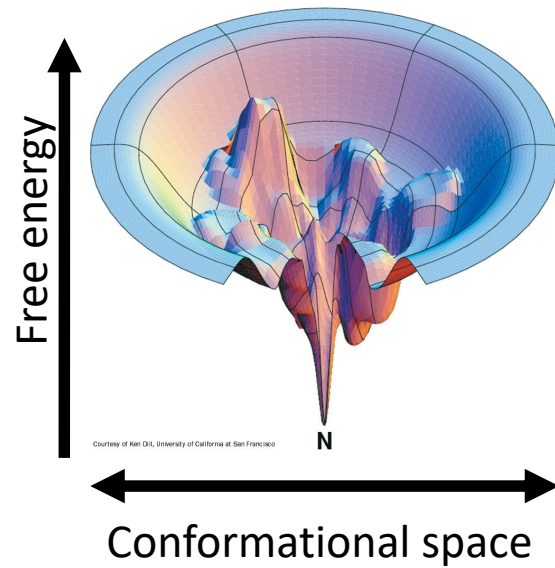


Modeling the protein free energy landscape

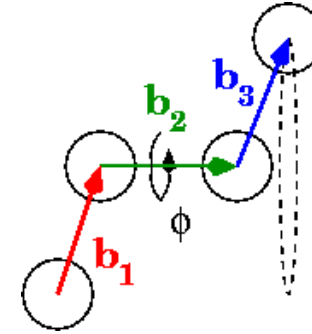


- Under *Anfinsen's hypothesis*, the state of lowest free energy is the native state
- Represent the various enthalpic and entropic effects governing folding with *parameterized equations*
 - vdW interactions
 - electrostatic interactions
 - solvent entropy
 - etc.
- **Predicting protein structure** involves identifying the lowest-energy state of the protein

Modeling the protein free energy landscape

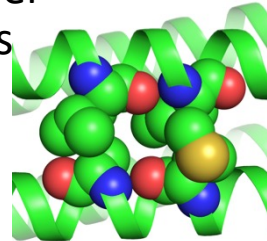


Bonded interactions

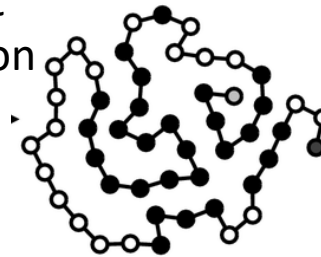


Non-bonded interactions

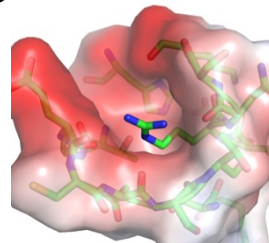
van der Waals



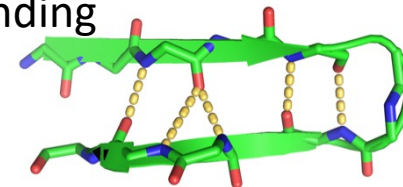
Implicit solvation



Electrostatics



Hydrogen bonding



Modeling covalent forces

Bond lengths

$$V_{bond} = K_b (b - b_0)^2$$

K_b = force constant

b_0 = equilibrium length

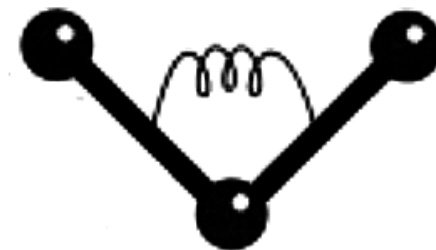
Chemical type	K_{bond}	b_0
C-C	100 kcal/mole/Å ²	1.5 Å
C=C	200 kcal/mole/Å ²	1.3 Å
C≡C	400 kcal/mole/Å ²	1.2 Å

Bond angle

$$V_{angle} = K_\theta (\theta - \theta_0)^2$$

K_θ = force constant

θ_0 = equilibrium angle



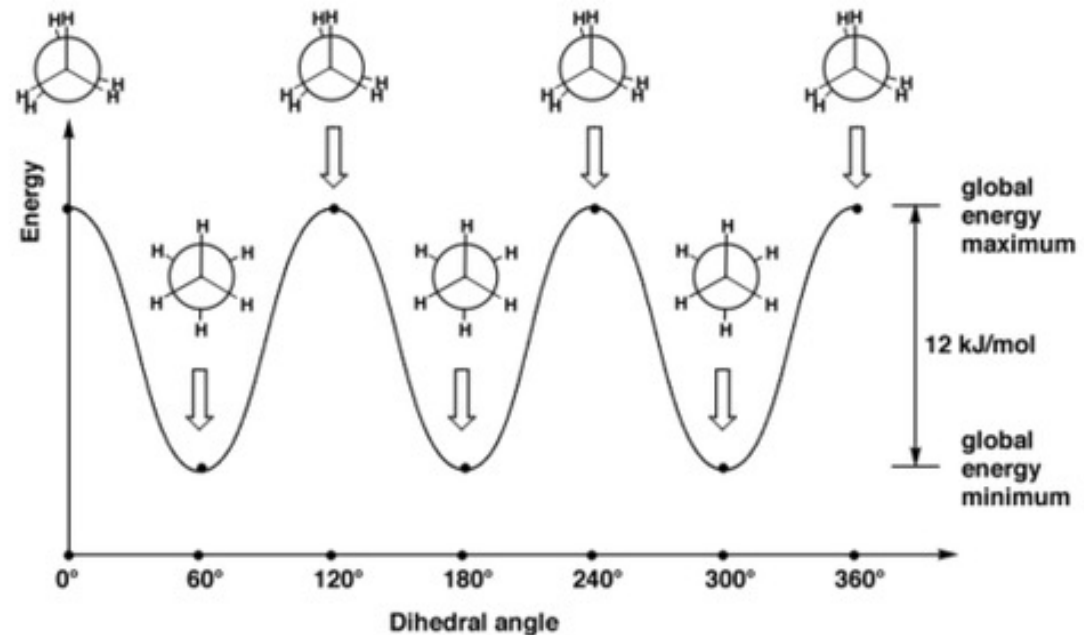
Modeling covalent forces

Torsion angle

- Staggered conformations (angle +60, -60 or 180 are preferred).



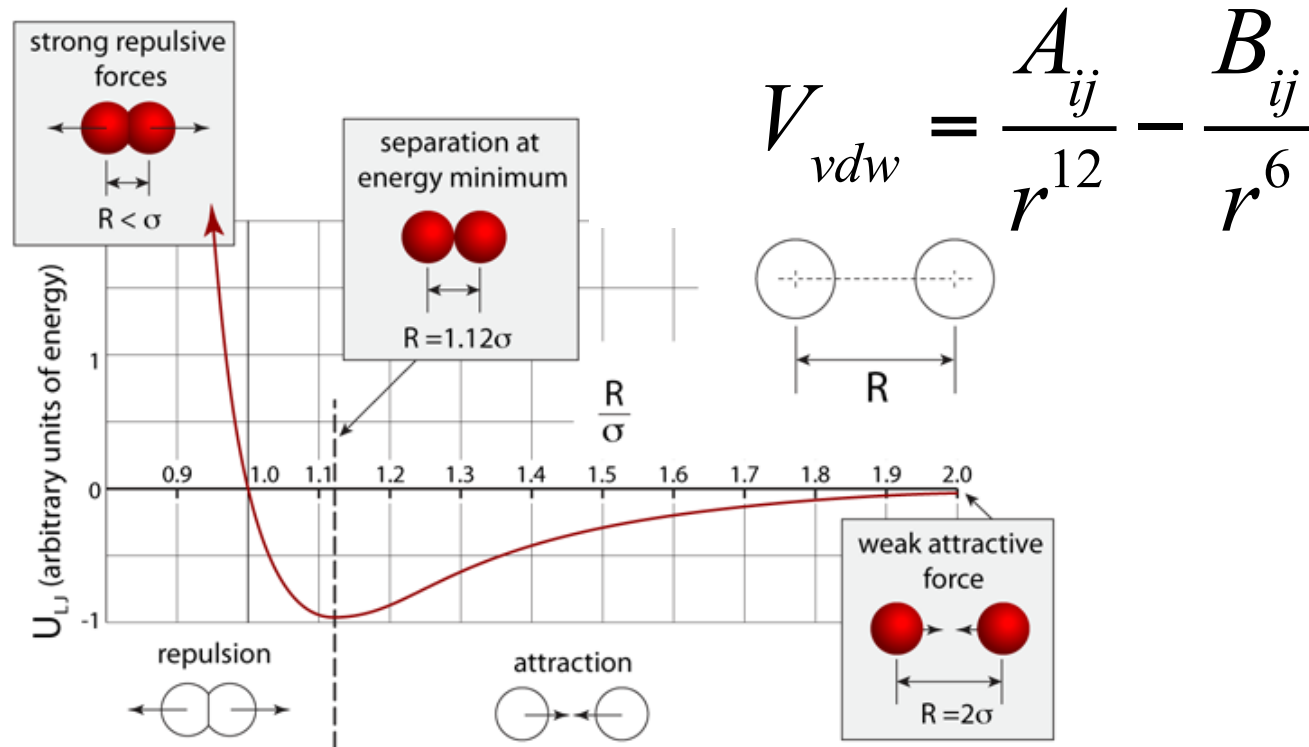
$$V_{torsion} = \sum_n k_n \cos(n\phi)$$



Nonbonded forces

Van der Waals forces

- Interactions between nonbonded atoms are expressed by the Lennard-Jones potential.
- Very high repulsive force if atoms closer than van der Waals radii; attractive force if distance greater

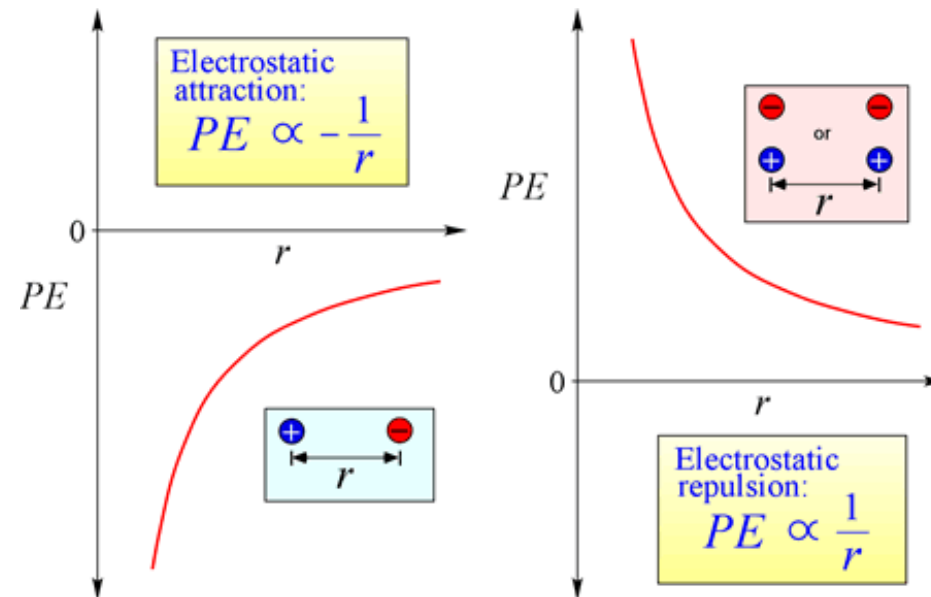


Nonbonded forces

Electrostatic interactions

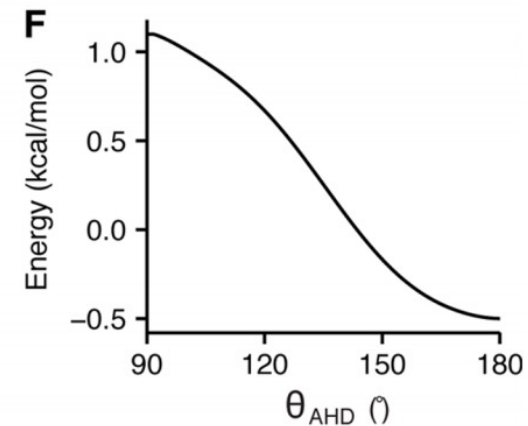
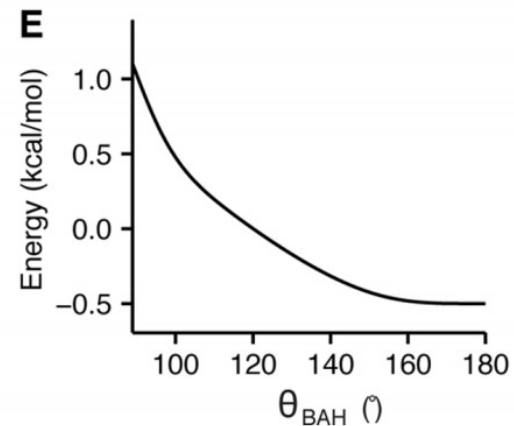
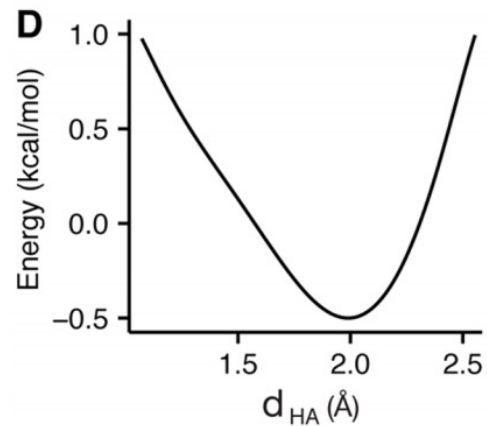
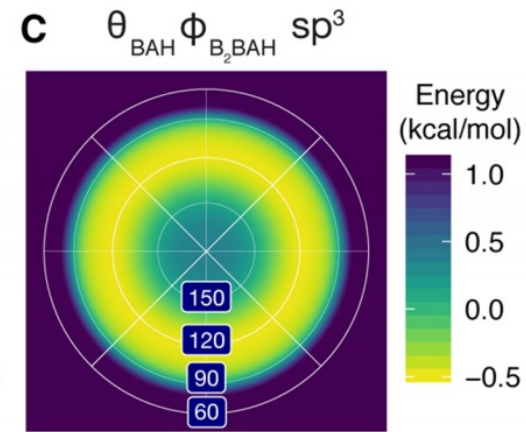
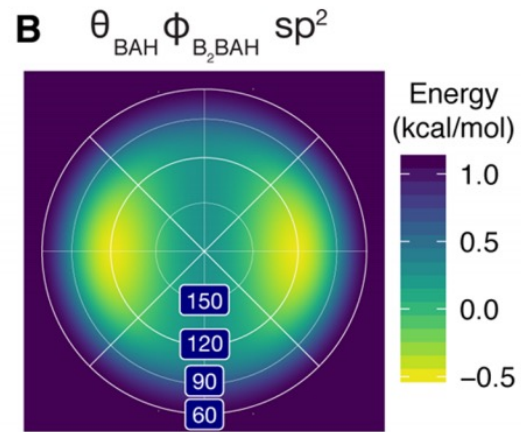
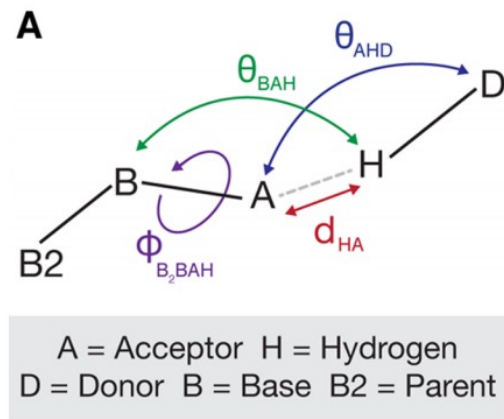
- Approximate dipoles by giving atoms a partial charge
- Dielectric constant varies according to media: $\epsilon=80$ for water, and 4-6?? in the core of protein
- Electrostatic energy falls off much less quickly than for van der Waals interactions (chemically significant at $\sim 15\text{\AA}$)

$$V_{\text{electrostatics}} = k_e \frac{q_1 q_2}{\epsilon r}$$



Nonbonded forces

Hydrogen bonding



Potential Energy

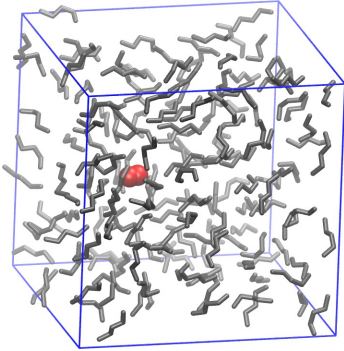
$$E_{\text{pot}} = \sum_b K_2 (b - b_0)^2 + \sum_{\theta} H_{\theta} (\theta - \theta_0)^2 + \sum_{\phi} \frac{V_n}{2} [1 + \cos(n\phi - \phi_0)]$$
$$+ \sum \epsilon [(r^*/r)^{12} - 2(r^*/r)^6] + \sum q_i q_j / \epsilon_{ij} r_{ij} + \sum \left[\frac{C_{ij}}{r_{ij}^{12}} - \frac{D_{ij}}{r_{ij}^{10}} \right]$$

(1) (2) (3) (4) (5) (6)

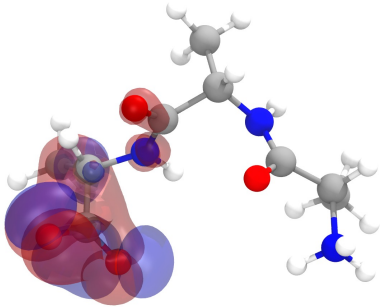
How is this useful?

- Compare **relative energies** of conformers
- Effect of **mutations** on stability
- Refining x-ray structures,
determining structures from NMR data

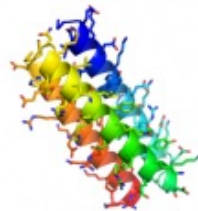
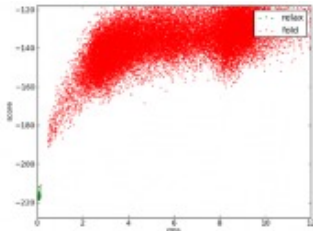
How are these functions parameterized?



... to match biophysical experiments on small molecules



... to match “higher level theory” simulations on small systems



... to maximize the ability to recapitulate structures/properties from protein crystal structures

Monte Carlo

In molecular simulations, Monte Carlo is an importance sampling technique

1. Make a random move and produce a new conformation
2. Calculate the energy change ΔE for the new conformation
3. Accept or reject the move based on the *Metropolis criterion*

$$P = \exp\left(-\frac{\Delta E}{kT}\right) \longrightarrow \text{Boltzmann factor}$$

If $\Delta E < 0$, then $P > 1$, accept new conformation;

Otherwise:

if $P > \text{rand}(0, 1)$, accept,
else reject.

Simulated Annealing Monte Carlo

In **Simulated Annealing Monte Carlo**, we reduce the temperature as the simulation progresses:

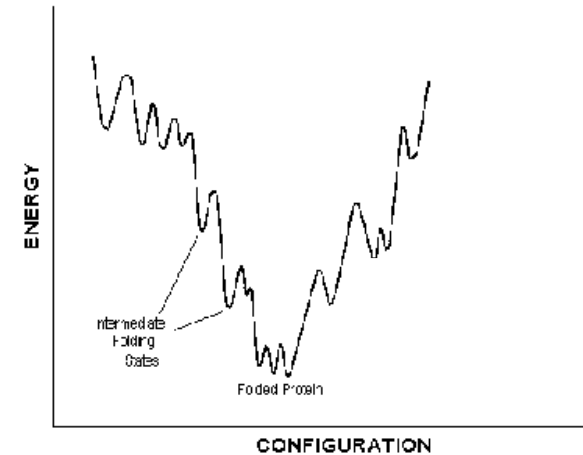
for $i=0:i_{max}$

$$T_k = (T_{max} - T_{min}) * (i_{max} - i) / i_{max} + T_{min}$$

Run k steps of Monte Carlo at temperature T_k

high T: accept almost all structures

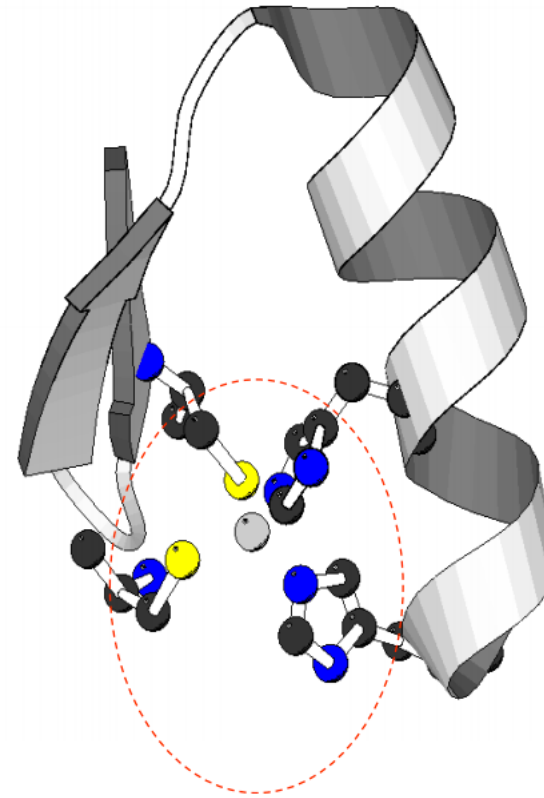
low T: accept almost only better structures



Example:

Sidechain rotamer determination

- **Problem:** given the backbone coordinates of a protein, predict the coordinates of the sidechain atoms
- Each sidechain has a discrete number of states (“rotamers”)
- Monte Carlo moves:
 - replace sidechain with random rotamer



Molecular Dynamics

Algorithm

- For atom i , Newton's equation of motion is given by

$$F_i = m_i a_i \quad \Rightarrow \quad \mathbf{F}_i(t) = m_i \frac{d^2 \mathbf{r}_i(t)}{dt^2}$$

Here, \mathbf{r}_i and m_i represent the position and mass of atom i and $\mathbf{F}_i(t)$ is the force on atom i at time t . $\mathbf{F}_i(t)$ can also be expressed as the gradient of the potential energy

$$\mathbf{F}_i = -\nabla_i V \quad \Rightarrow \quad -\nabla_i V = m_i \frac{d^2 \mathbf{r}_i(t)}{dt^2}$$

V is potential energy. Newton's equation of motion can then relate the derivative of the potential energy to the changes in position as a function of time.

Molecular Dynamics

Numeric integration by using the **Verlet algorithm**

- Given initial velocity 0 and position x_i , numerically integrate to get position at time $t+\delta t$
- Taylor expansions to 3rd order for i

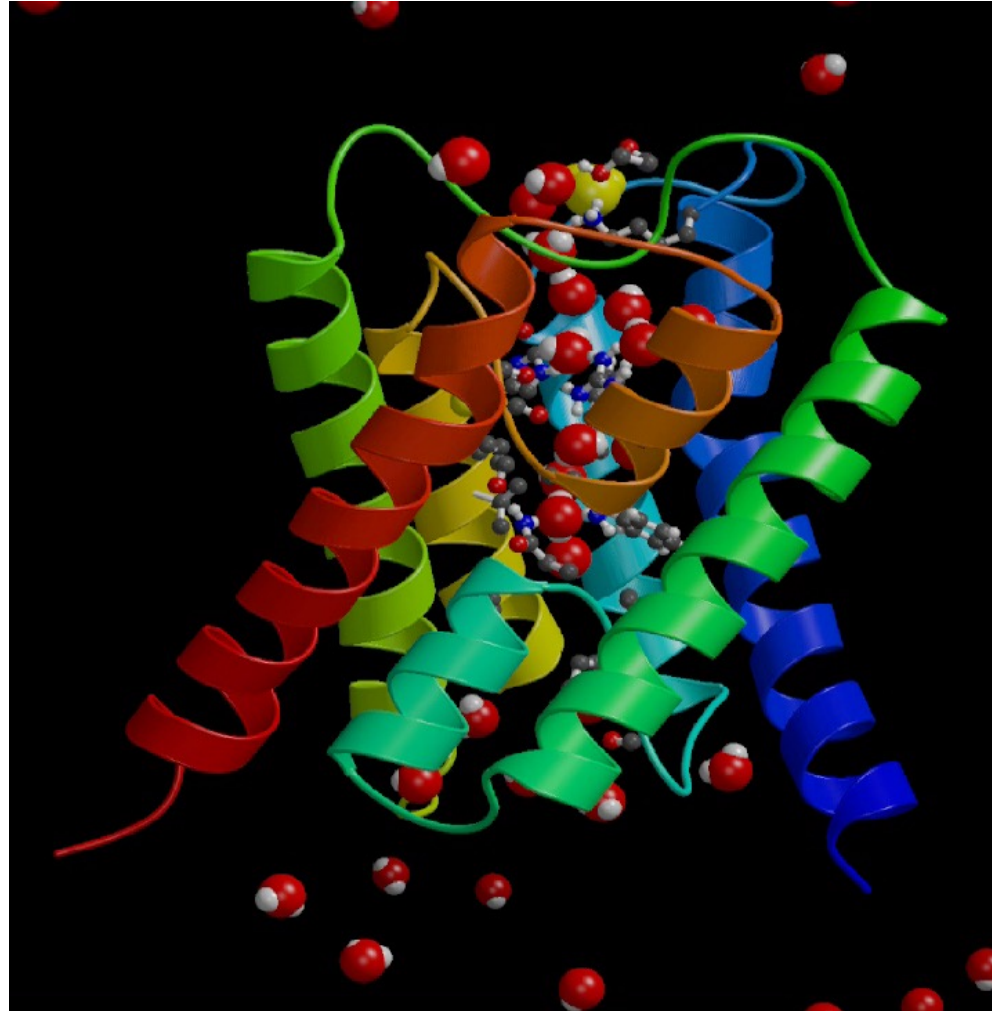
$$\mathbf{r}(t + \delta t) = \mathbf{r}(t) + (\delta t)\mathbf{v}(t) + \frac{1}{2}(\delta t)^2 \mathbf{a}(t) + \frac{1}{6}(\delta t)^3 \mathbf{b}(t) + \dots$$

$$\mathbf{r}(t - \delta t) = \mathbf{r}(t) - (\delta t)\mathbf{v}(t) + \frac{1}{2}(\delta t)^2 \mathbf{a}(t) - \frac{1}{6}(\delta t)^3 \mathbf{b}(t) + \dots$$

- Adding these equations gives [up to order $(\delta t)^4$]:

$$\mathbf{r}(t + \delta t) = 2\mathbf{r}(t) - \mathbf{r}(t - \delta t) + (\delta t)^2 \mathbf{a}(t) + O[(\delta t)^4]$$

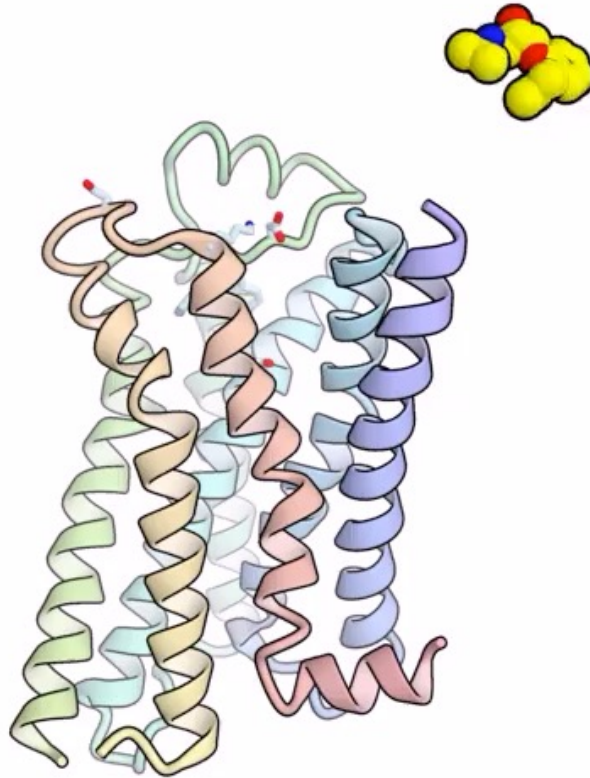
Aquaporin-1



(B.L. de Groot and H. Grubmüller: Science 294, 2353-2357 (2001))

Drug binding to GPCRs

0.00 us



Dror et al., PNAS 2011